

“Hamiltonian Quantum Cellular Automata in 1D”
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Outline

1. Introduction
2. Universal HQCA with $d = 10$
3. Universal HQCA with $d = 20$
4. Conclusion

HQCA – Hamiltonian Quantum cellular automaton

Universal – can be used to solve all problems in BQP

d – the dimension of the state space of each cell

Introduction

Hamiltonian Quantum cellular automaton

How it works?

1. Take a rotating supermassive *black hole*
2. Shine a certain amount of the *dark energy* into it
3. Measure the change of the *cosmological constant* with respect to the Friedmann-Lemaître-Robertson-Walker metric

Hamiltonian Quantum cellular automaton

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I'm joking...

Hamiltonian Quantum cellular automaton

How it works?

1. Prepare a basis state containing both – the *input* and the *program* itself
2. Let this state evolve undisturbed according to some *fixed* Hamiltonian for some amount of time
3. Measure a small subsystem in the computational basis to get the answer

Motivation

For me

I had to choose some topic

Motivation

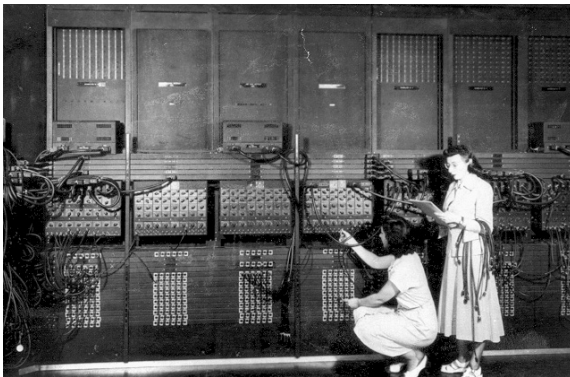
For me

It's a nice idea – reminds me of *von Neumann architecture*, *universal Turing machine* and ENIAC. . .

Motivation

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It's a nice idea – reminds me of *von Neumann architecture*, *universal Turing machine* and ENIAC. . .



Two women wiring the right side of the ENIAC with a new program, in the “pre-von Neumann” days. Standing: Ester Gerston, Crouching: Gloria Ruth Gordon.

Main ingredients

1. Universal set of quantum gates
2. Classical cellular automaton
3. Quantum walk (for required time analysis)

Real universal gates

Theorem (Rudolph & Grover, 2002)

The gate

$$G(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \phi & -\sin \phi \\ 0 & 0 & \sin \phi & \cos \phi \end{pmatrix}$$

is universal for quantum computing, when ϕ is an irrational multiple of π . [RG02]

Idea: Simulate one-qubit gates and CNOT, which are universal according to [BBC+95].

Real universal gates

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Note

We can apply G to any pair of qubits. We have to:

- ▶ simulate the real and imaginary parts separately,
- ▶ use ancilla qubits.

Thus we get universality only in some subspace.

The W gate

Claim (Nagaj & Wocjan, 2008)

The gate

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = G\left(\frac{\pi}{4}\right)$$

is universal for quantum computing. [NW08]

Idea: Simulate Toffoli and Hadamard gates, which are universal according to [Aha03]. **How?**

The W gate

Claim (Nagaj & Wocjan, 2008)

The gate

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Idea: Simulate Toffoli and Hadamard gates, which are universal according to [Aha03]. **How?**

Swapping qubits

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

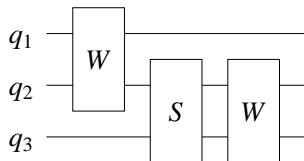
The main challenge

We would like our Hamiltonian to be as simple as possible:

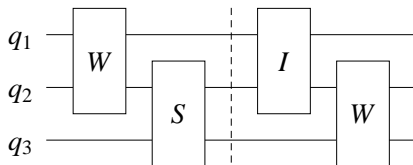
- ▶ it acts on a 1D chain of qudits,
- ▶ it is local – only nearest-neighbor interactions,
- ▶ it is translationally invariant,
- ▶ qudits have as small dimension as possible.

Circuits in “ladder form”

Original circuit



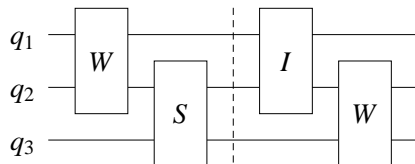
Modified circuit



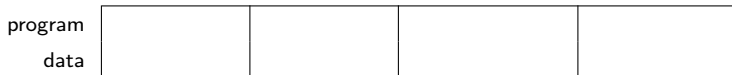
The first construction ($d = 10$)

Encoding the circuit

Circuit in “ladder form”

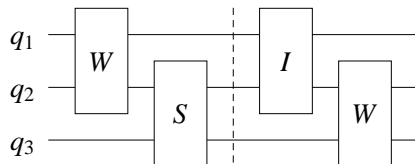


Encoded circuit

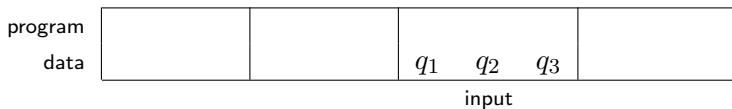


Encoding the circuit

Circuit in “ladder form”

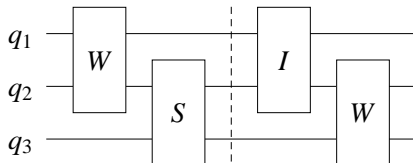


Encoded circuit

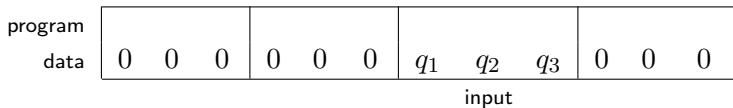


Encoding the circuit

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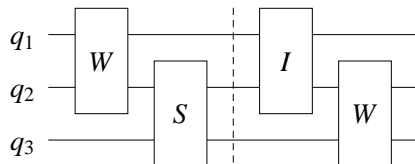


Encoded circuit



Encoding the circuit

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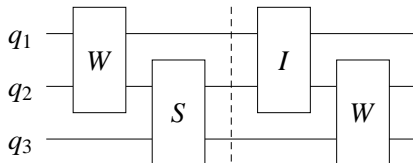


Encoded circuit

		1st sequence											
program								W	S				
data		0	0	0	0	0	0	q_1	q_2	q_3	0	0	0
		input											

Encoding the circuit

Circuit in “ladder form”

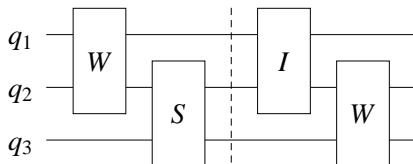


Encoded circuit

							1st sequence		2nd sequence			
program							W	S	I	W		
data	0	0	0	0	0	0	q_1	q_2	q_3	0	0	0
	input											

Encoding the circuit

Circuit in “ladder form”

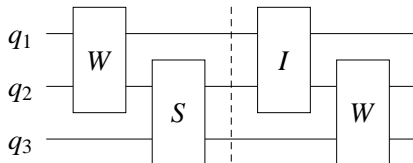


Encoded circuit

				1st sequence			2nd sequence					
program							<i>I</i>	<i>W</i>	<i>S</i>	<i>I</i>	<i>I</i>	<i>W</i>
data	0	0	0	0	0	0	q_1	q_2	q_3	0	0	0
	input											

Encoding the circuit

Circuit in “ladder form”

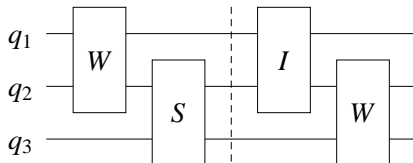


Encoded circuit

				1st marker	1st sequence			2nd sequence				
program				•	•	▶	I	W	S	I	I	W
data	0	0	0	0	0	0	q_1	q_2	q_3	0	0	0
	input											

Encoding the circuit

Circuit in “ladder form”

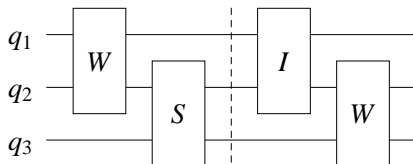


Encoded circuit

	2nd marker			1st marker			1st sequence			2nd sequence		
program	•	•	▶	•	•	▶	I	W	S	I	I	W
data	0	0	0	0	0	0	q_1	q_2	q_3	0	0	0
	input											

Encoding the circuit

Circuit in “ladder form”



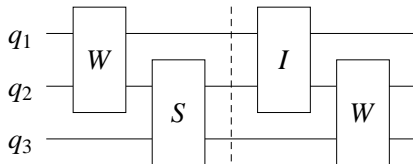
Encoded circuit

	2nd marker			1st marker			1st sequence			2nd sequence		
program	•	•	▶	•	•	▶	I	W	S	I	I	W
data	0	0	0	0	0	0	q_1	q_2	q_3	0	0	0
	input											

Basis states are marked by elements of $\{\bullet, \blacktriangleright, W, S, I\} \times \{0, 1\}$.

Evolution

Circuit

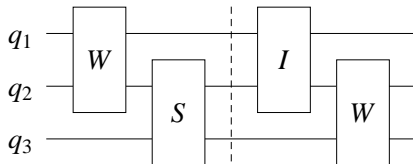


Current configuration

•	•	▶	•	•	▶	I	W	S	I	I	W
0	0	0	0	0	0	q_1	q_2	q_3	0	0	0

Evolution

Circuit

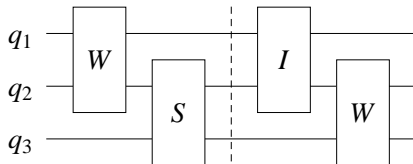


Current configuration

•	•	▶	•	•	I	▶	W	S	I	I	W
0	0	0	0	0	0	q_1	q_2	q_3	0	0	0

Evolution

Circuit

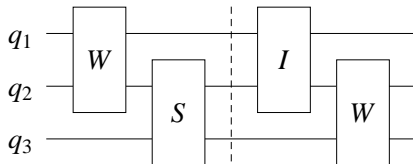


Current configuration

\cdot	\cdot	\blacktriangleright	\cdot	\cdot	I	W	\blacktriangleright	S	I	I	W
0	0	0	0	0	0	$ q_{12}\rangle$		q_3	0	0	0

Evolution

Circuit

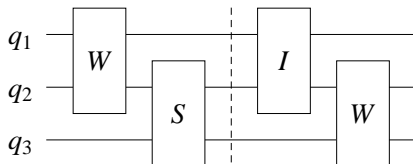


Current configuration

\cdot	\cdot	\blacktriangleright	\cdot	\cdot	I	W	S	\blacktriangleright	I	I	W
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

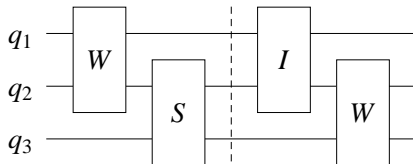


Current configuration

\cdot	\cdot	\blacktriangleright	\cdot	\cdot	I	W	S	I	\blacktriangleright	I	W
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

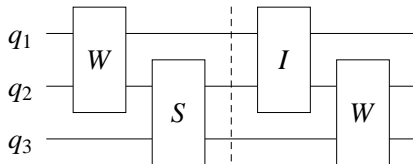


Current configuration

\cdot	\cdot	\blacktriangleright	\cdot	\cdot	I	W	S	I	I	\blacktriangleright	W
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

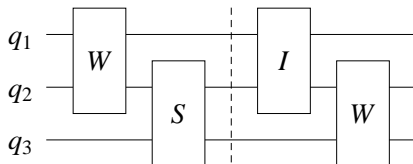


Current configuration

\cdot	\cdot	\blacktriangleright	\cdot	\cdot	I	W	S	I	I	W	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

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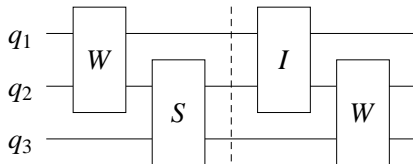


Current configuration

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0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

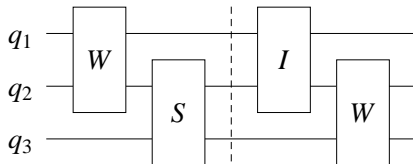


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0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

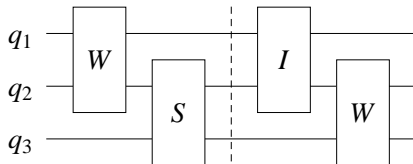


Current configuration

\cdot	\cdot	\blacktriangleright	I	\cdot	W	\cdot	S	I	I	W	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

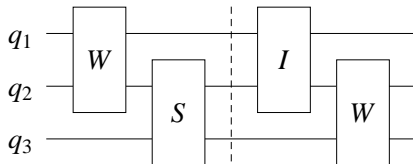


Current configuration

•	•	I	▶	•	W	•	S	I	I	W	▶
0	0	0	0	0	0		q_{123}	⟩	0	0	0

Evolution

Circuit

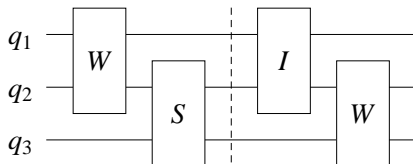


Current configuration

\cdot	\cdot	I	\blacktriangleright	\cdot	W	S	\cdot	I	I	W	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

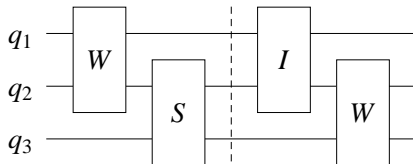


Current configuration

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0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

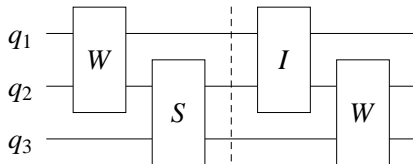


Current configuration

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0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

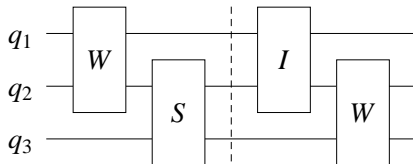


Current configuration

\cdot	\cdot	I	\blacktriangleright	\cdot	W	S	I	I	W	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

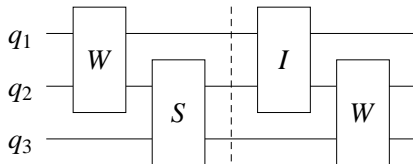


Current configuration

\cdot	\cdot	I	\blacktriangleright	W	\cdot	S	I	I	W	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

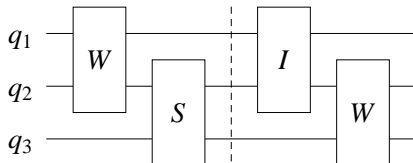


Current configuration

\cdot	\cdot	I	\blacktriangleright	W	S	\cdot	I	I	W	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

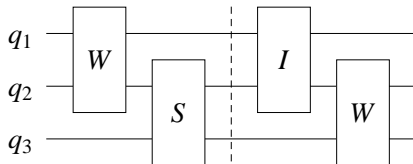


Current configuration

\cdot	\cdot	I	\blacktriangleright	W	S	I	\cdot	I	W	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

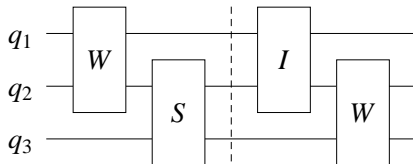


Current configuration

\cdot	\cdot	I	\blacktriangleright	W	S	I	I	\cdot	W	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

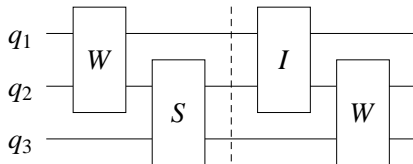


Current configuration

\cdot	\cdot	I	\blacktriangleright	W	S	I	I	W	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

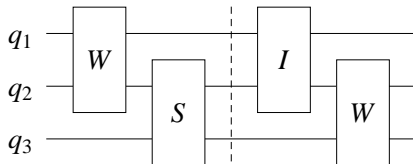


Current configuration

\cdot	\cdot	I	W	\blacktriangleright	S	I	I	W	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

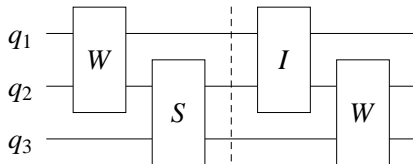


Current configuration

\cdot	\cdot	I	W	S	\blacktriangleright	I	I	W	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

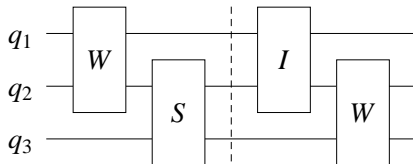


Current configuration

\cdot	\cdot	I	W	S	I	\blacktriangleright	I	W	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

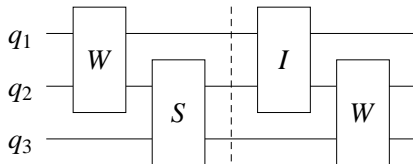


Current configuration

\cdot	\cdot	I	W	S	I	I	\blacktriangleright	W	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		q_{123}	\rangle	0	0	0

Evolution

Circuit

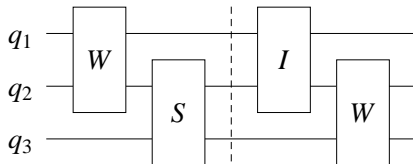


Current configuration

\cdot	\cdot	I	W	S	I	I	W	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

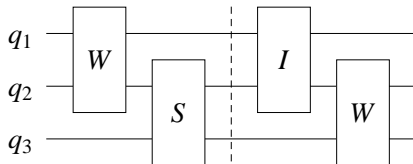


Current configuration

\cdot	I	\cdot	W	S	I	I	W	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

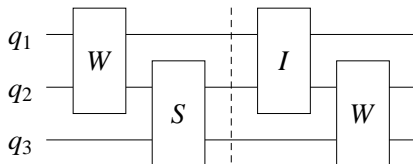


Current configuration

\cdot	I	W	\cdot	S	I	I	W	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

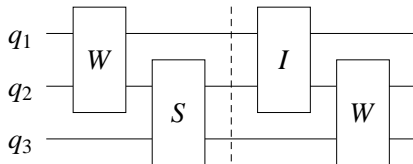


Current configuration

\cdot	I	W	S	\cdot	I	I	W	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

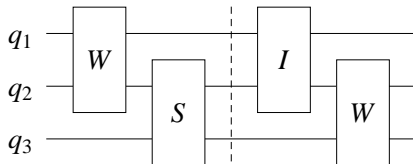


Current configuration

\cdot	I	W	S	I	\cdot	I	W	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

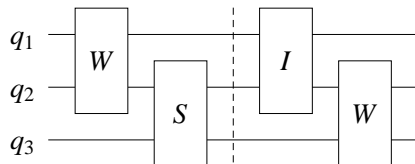


Current configuration

\cdot	I	W	S	I	I	\cdot	W	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0	$ $	s_{123}	\rangle	0	0	0

Evolution

Circuit

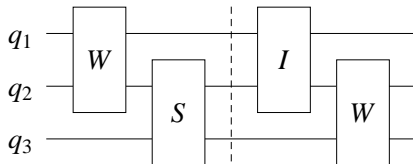


Current configuration

.	I	W	S	I	I	W	.	▶	.	.	▶
0	0	0	0	0	0		s_{123}	⟩	0	0	0

Evolution

Circuit

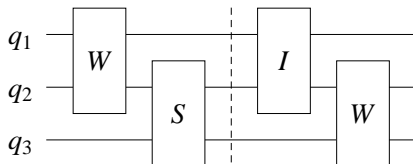


Current configuration

I	\cdot	W	S	I	I	W	\cdot	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

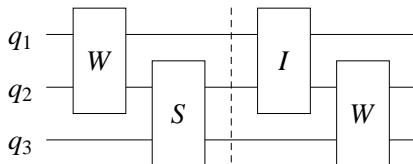


Current configuration

I	W	\cdot	S	I	I	W	\cdot	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

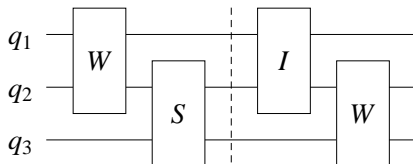


Current configuration

I	W	S	\cdot	I	I	W	\cdot	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit

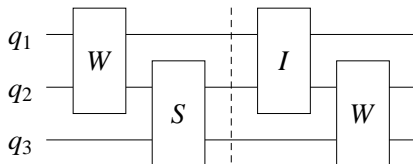


Current configuration

I	W	S	I	.	I	W	.	▶	.	.	▶
0	0	0	0	0	0		s_{123}	⟩	0	0	0

Evolution

Circuit

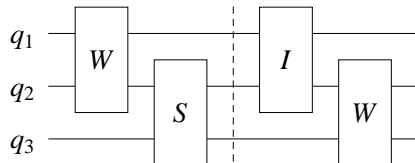


Current configuration

I	W	S	I	I	\cdot	W	\cdot	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Evolution

Circuit



Current configuration

I	W	S	I	I	W	\cdot	\cdot	\blacktriangleright	\cdot	\cdot	\blacktriangleright
0	0	0	0	0	0		s_{123}	\rangle	0	0	0

Rules

Move gate

$$\begin{array}{|c|c|} \hline \cdot & A \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} \hline A & \cdot \\ \hline \end{array} \quad \text{where } A \in \{W, S, I\}$$

Apply gate

$$\begin{array}{|c|c|} \hline \blacktriangleright & A \\ \hline x & y \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} \hline A & \blacktriangleright \\ \hline A(x, y) \\ \hline \end{array} \quad \text{where } A \in \{W, S, I\}$$

Hamiltonian

These rules give us

$$H_{10} = - \sum_{j=1}^{L-1} (R + R^\dagger)_{(j,j+1)}$$

$$R = \sum_{A \in \{W, S, I\}} \left(|A \cdot \rangle \langle \cdot A| \otimes I + |A \blacktriangleright \rangle \langle \blacktriangleright A| \otimes A \right)$$

Hamiltonian

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Consider how H_{10} acts on the program register. If we map $\{\blacktriangleright, \cdot\}$ to $|\bar{0}\rangle$ (no fermion) and $\{W, S, I\}$ to $|\bar{1}\rangle$ (a fermion), then we get

$$H_{\text{fermion}} = - \sum_{j=1}^{L-1} \left(|\bar{1}\bar{0}\rangle \langle \bar{0}\bar{1}| + |\bar{0}\bar{1}\rangle \langle \bar{1}\bar{0}| \right)_{(j,j+1)}$$

or *diffusion of a discrete free fermion gas on a line!*

Hamiltonian

These rules give us

$$H_{10} = - \sum_{j=1}^{L-1} (R + R^\dagger)_{(j,j+1)}$$

$$R = \sum_{A \in \{W, S, I\}} \left(|A \cdot \rangle \langle \cdot A| \otimes I + |A \blacktriangleright \rangle \langle \blacktriangleright A| \otimes A \right)$$

Consider how H_{10} acts on the program register. If we map $\{\blacktriangleright, \cdot\}$ to $|\bar{0}\rangle$ (no fermion) and $\{W, S, I\}$ to $|\bar{1}\rangle$ (a fermion), then we get

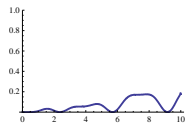
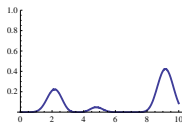
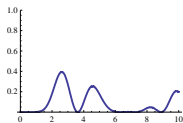
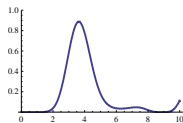
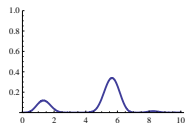
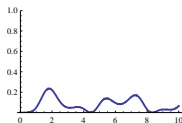
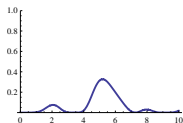
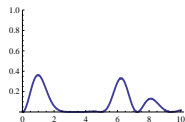
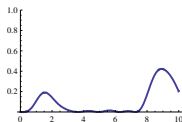
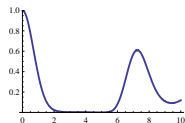
$$H_{\text{fermion}} = - \sum_{j=1}^{L-1} \left(|\bar{1}\bar{0}\rangle \langle \bar{0}\bar{1}| + |\bar{0}\bar{1}\rangle \langle \bar{1}\bar{0}| \right)_{(j,j+1)}$$

or *diffusion of a discrete free fermion gas on a line!* In other words: the initial state remains in the subspace spanned by the basis vectors of the same Hamming weight – fermions jumping around.

“Fermion love story”

Example with 5 sites and 2 fermions... ♡

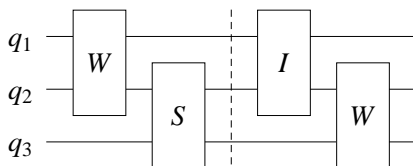
$|00011\rangle$
 $|00110\rangle$ $|00101\rangle$
 $|01100\rangle$ $|01010\rangle$ $|01001\rangle$
 $|11000\rangle$ $|10100\rangle$ $|10010\rangle$ $|10001\rangle$



The second construction ($d = 20$)

Encoding the circuit

Circuit in “ladder form”



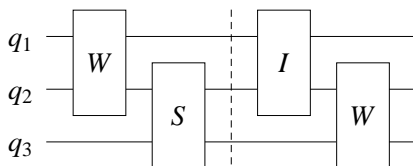
Encoded circuit

program

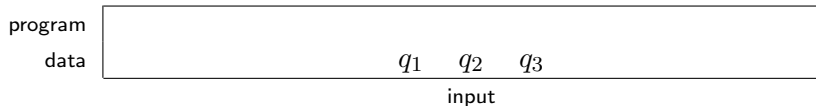
data

Encoding the circuit

Circuit in “ladder form”

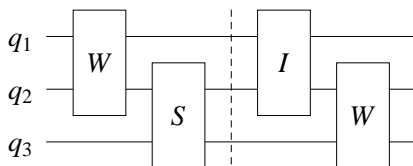


Encoded circuit

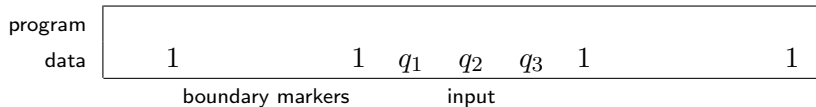


Encoding the circuit

Circuit in “ladder form”

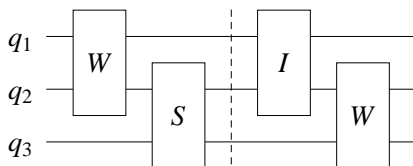


Encoded circuit

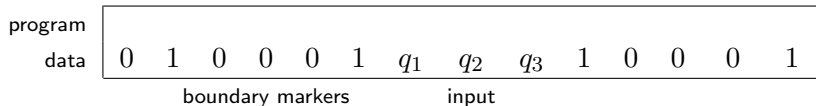


Encoding the circuit

Circuit in “ladder form”

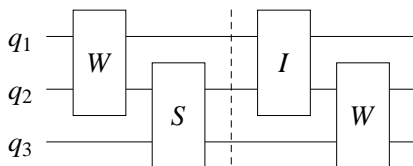


Encoded circuit

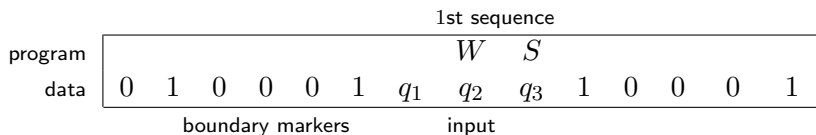


Encoding the circuit

Circuit in “ladder form”

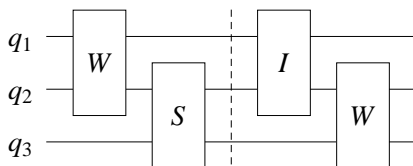


Encoded circuit



Encoding the circuit

Circuit in “ladder form”

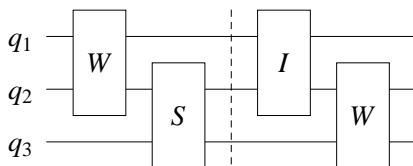


Encoded circuit

							1st sequence			2nd sequence				
program							W	S		I	W			
data	0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1
	boundary markers						input							

Encoding the circuit

Circuit in “ladder form”

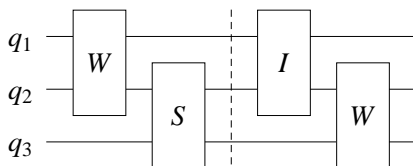


Encoded circuit

							1st sequence			2nd sequence				
program							I	W	S	I	I	I	W	
data	0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1
	boundary markers						input							

Encoding the circuit

Circuit in “ladder form”

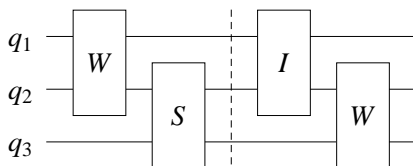


Encoded circuit

							1st sequence			2nd sequence					
program							I	W	S	I	I	I	W	\odot	
data	0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1	
	boundary markers						input								

Encoding the circuit

Circuit in “ladder form”

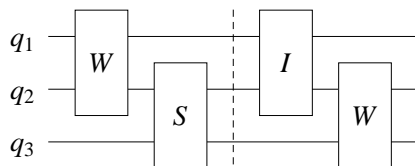


Encoded circuit

							1st sequence			2nd sequence				
program	•	•	•	•	•	•	I	W	S	I	I	I	W	⊙
data	0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1
	boundary markers						input							

Encoding the circuit

Circuit in “ladder form”



Encoded circuit

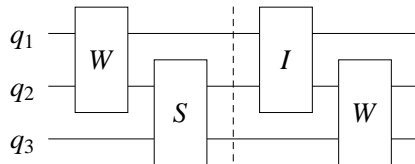
							1st sequence			2nd sequence				
program	•	•	•	•	•	•	I	W	S	I	I	I	W	\odot
data	0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1
	boundary markers						input							

Basis states are marked by elements of

$$\{W, S, I, \textcircled{W}, \textcircled{S}, \textcircled{I}, \blacktriangleright, \triangleright, \odot, \bullet\} \times \{0, 1\}.$$

Evolution

Circuit

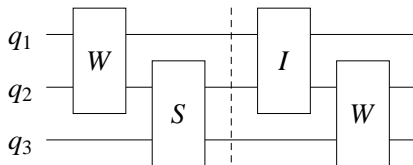


Current configuration

•	•	•	•	•	•	I	W	S	I	I	I	W	↻
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

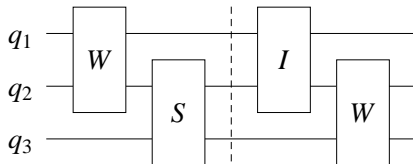


Current configuration

•	•	•	•	•	•	I	W	S	I	I	I	⊗ W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

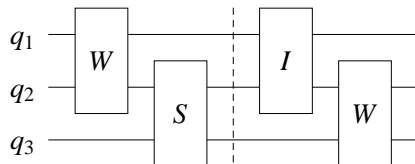


Current configuration

•	•	•	•	•	•	I	W	S	I	I	\textcircled{I}	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

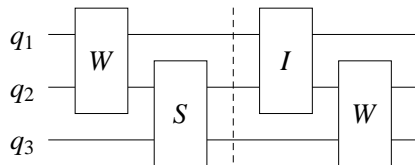


Current configuration

•	•	•	•	•	•	I	W	S	I	\textcircled{I}	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

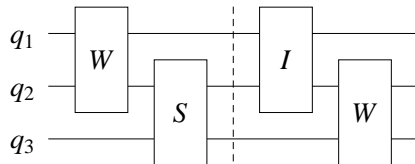


Current configuration

•	•	•	•	•	•	I	W	S	\textcircled{I}	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

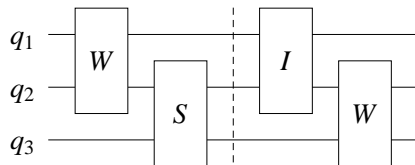


Current configuration

•	•	•	•	•	•	I	W	Ⓢ	I	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

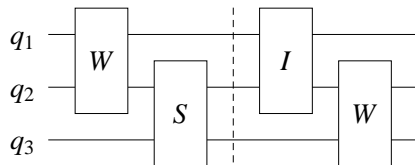


Current configuration

•	•	•	•	•	•	I	Ⓜ	S	I	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

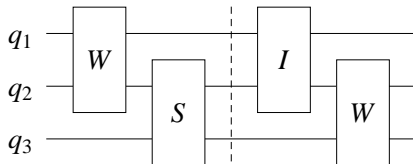


Current configuration

•	•	•	•	•	•	Ⓢ	W	S	I	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

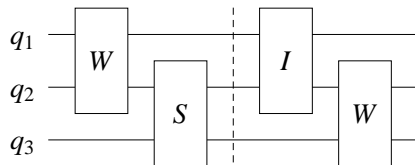


Current configuration

•	•	•	•	•	⊙	I	W	S	I	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

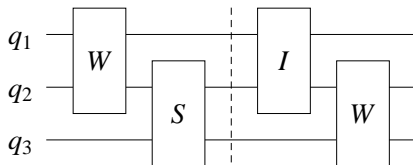


Current configuration

•	•	•	•	•	⊙	I	W	S	I	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

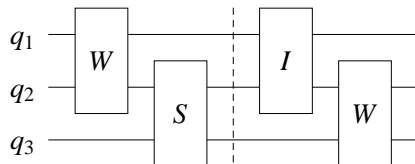


Current configuration

•	•	•	•	•	▶	<i>I</i>	<i>W</i>	<i>S</i>	<i>I</i>	<i>I</i>	<i>I</i>	<i>W</i>	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

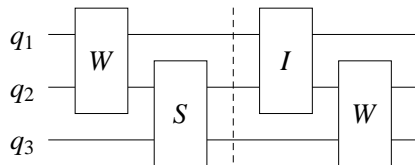


Current configuration

•	•	•	•	•	▶	I	W	S	I	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

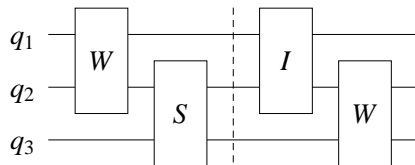


Current configuration

•	•	•	•	•	I	▶	W	S	I	I	I	W	•
0	1	0	0	0	1	q_1	q_2	q_3	1	0	0	0	1

Evolution

Circuit

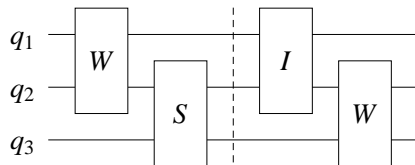


Current configuration

•	•	•	•	•	I	W	▶	S	I	I	I	W	•
0	1	0	0	0	1	$ q_{12}\rangle$		q_3	1	0	0	0	1

Evolution

Circuit

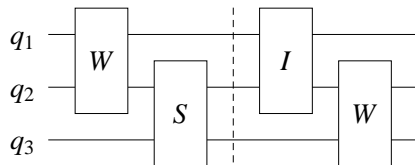


Current configuration

•	•	•	•	•	I	W	S	▶	I	I	I	W	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

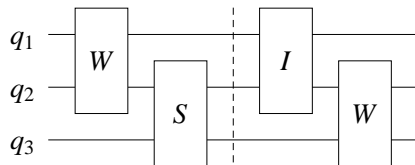


Current configuration

•	•	•	•	•	I	W	S	I	▶	I	I	W	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

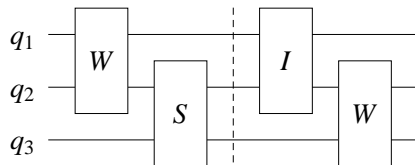


Current configuration

$\begin{array}{ccccccccccccccc}
 \cdot & \cdot & \cdot & \cdot & \cdot & I & W & S & I & I & \blacktriangleright & I & W & \cdot \\
 0 & 1 & 0 & 0 & 0 & 1 & | & q_{123} & \rangle & 1 & 0 & 0 & 0 & 1
 \end{array}$

Evolution

Circuit

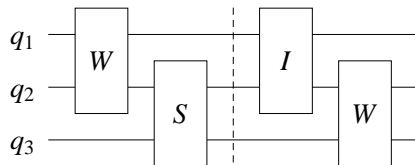


Current configuration

$\begin{array}{ccccccccccccccc}
 \bullet & \bullet & \bullet & \bullet & \bullet & I & W & S & I & I & I & \blacktriangleright & W & \bullet \\
 0 & 1 & 0 & 0 & 0 & 1 & | & q_{123} & \rangle & 1 & 0 & 0 & 0 & 1
 \end{array}$

Evolution

Circuit

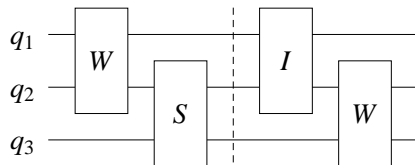


Current configuration

•	•	•	•	•	I	W	S	I	I	I	W	▶	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

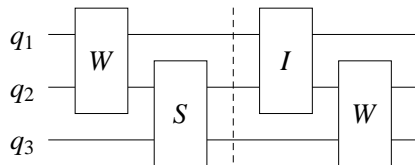


Current configuration

\cdot \cdot \cdot \cdot \cdot I W S I I I W \blacktriangleright \cdot
 0 1 0 0 0 1 | q_{123} \rangle 1 0 0 0 0 **1**

Evolution

Circuit

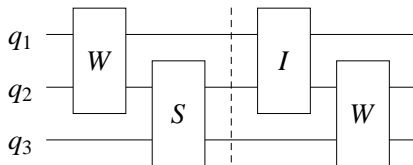


Current configuration

•	•	•	•	•	I	W	S	I	I	I	W	↻	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

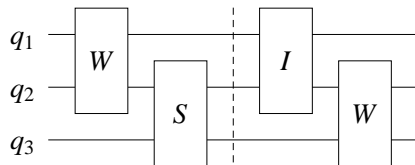


Current configuration

•	•	•	•	•	I	W	S	I	I	I	W	⊙	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

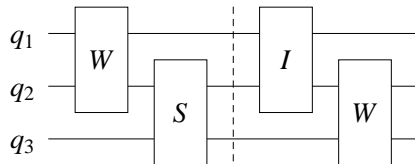


Current configuration

•	•	•	•	•	I	W	S	I	I	I	\textcircled{W}	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

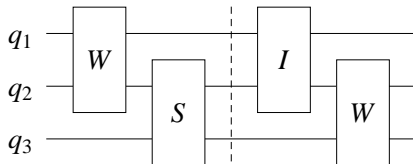


Current configuration

•	•	•	•	•	I	W	S	I	I	\textcircled{I}	W	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

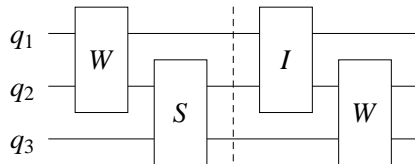


Current configuration

$\begin{array}{cccccccccccccccc}
 \cdot & \cdot & \cdot & \cdot & \cdot & I & W & S & I & \textcircled{I} & I & W & \cdot & \cdot \\
 0 & 1 & 0 & 0 & 0 & 1 & | & q_{123} & \rangle & 1 & 0 & 0 & 0 & 1
 \end{array}$

Evolution

Circuit

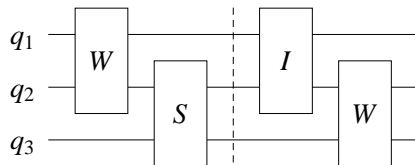


Current configuration

•	•	•	•	•	I	W	S	\textcircled{I}	I	I	W	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

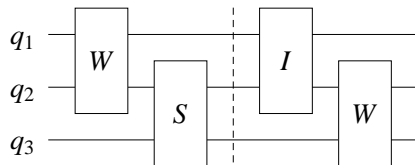


Current configuration

$\begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot & I & W & \textcircled{S} & I & I & I & W & \cdot & \cdot \\ 0 & 1 & 0 & 0 & 0 & 1 & | & q_{123} & \rangle & 1 & 0 & 0 & 0 & 1 \end{matrix}$

Evolution

Circuit

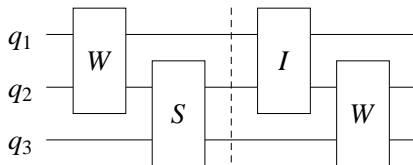


Current configuration

•	•	•	•	•	I	⊗	S	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

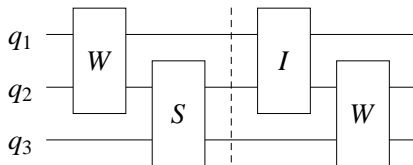


Current configuration

•	•	•	•	•	\textcircled{I}	W	S	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

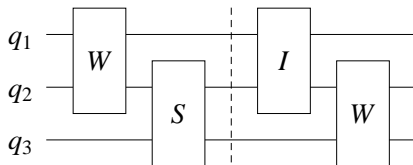


Current configuration

•	•	•	•	⊙	I	W	S	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

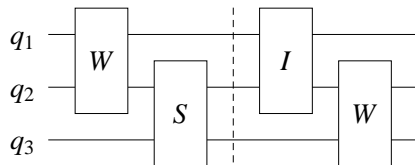


Current configuration

•	•	•	•	⊙	I	W	S	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

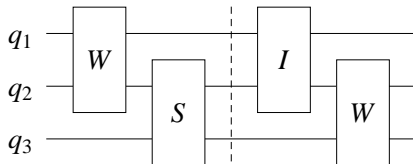


Current configuration

•	•	•	•	▷	I	W	S	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

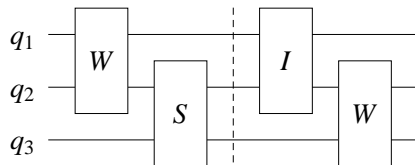


Current configuration

•	•	•	•	▷	I	W	S	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

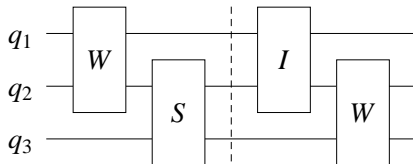


Current configuration

•	•	•	•	I	▷	W	S	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

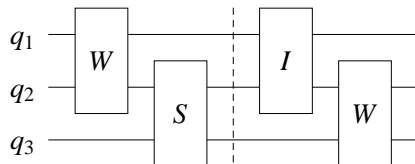


Current configuration

$$\begin{array}{cccccccccccccccc}
 \cdot & \cdot & \cdot & \cdot & I & W & \triangleright & S & I & I & I & W & \cdot & \cdot \\
 0 & 1 & 0 & 0 & 0 & 1 & | & q_{123} & \rangle & 1 & 0 & 0 & 0 & 1
 \end{array}$$

Evolution

Circuit

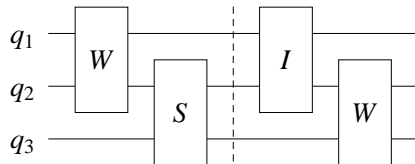


Current configuration

•	•	•	•	I	W	S	\triangleright	I	I	I	W	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

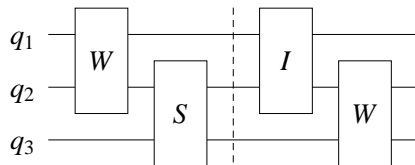


Current configuration

•	•	•	•	I	W	S	I	\triangleright	I	I	W	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

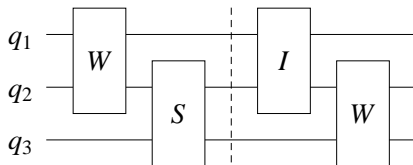


Current configuration

•	•	•	•	I	W	S	I	I	\triangleright	I	W	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

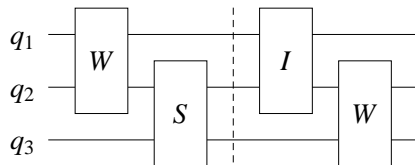


Current configuration

•	•	•	•	I	W	S	I	I	I	▷	W	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

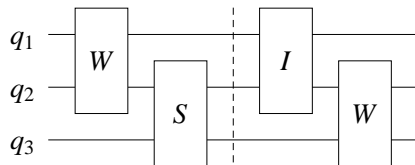


Current configuration

•	•	•	•	I	W	S	I	I	I	W	\triangleright	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

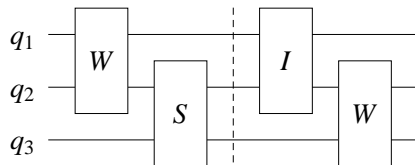


Current configuration

•	•	•	•	<i>I</i>	<i>W</i>	<i>S</i>	<i>I</i>	<i>I</i>	<i>I</i>	<i>W</i>	▷	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

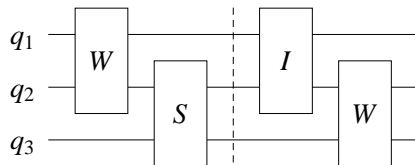


Current configuration

•	•	•	•	I	W	S	I	I	I	W	⊙	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

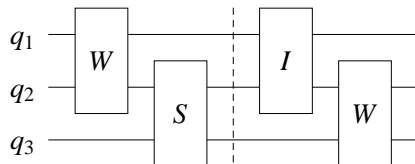


Current configuration

•	•	•	•	I	W	S	I	I	I	W	⊙	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

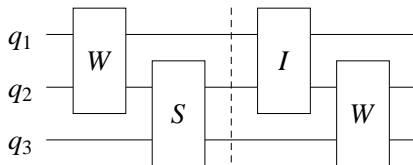


Current configuration

•	•	•	•	I	W	S	I	I	I	Ⓜ	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

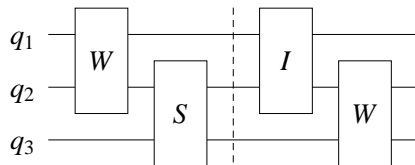


Current configuration

\cdot	\cdot	\cdot	\cdot	I	W	S	I	I	\textcircled{I}	W	\cdot	\cdot	\cdot
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

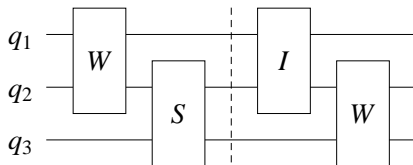


Current configuration

•	•	•	•	I	W	S	I	\textcircled{I}	I	W	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

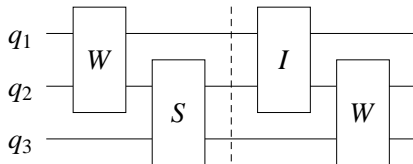


Current configuration

\cdot	\cdot	\cdot	\cdot	I	W	S	\textcircled{I}	I	I	W	\cdot	\cdot	\cdot
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

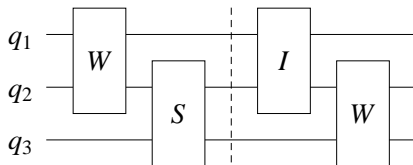


Current configuration

•	•	•	•	I	W	⊙	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

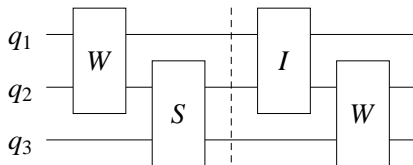


Current configuration

•	•	•	•	I	Ⓜ	S	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

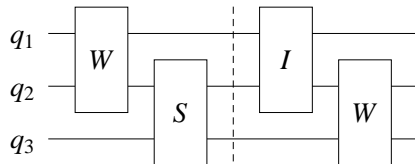


Current configuration

•	•	•	•	Ⓢ	<i>W</i>	<i>S</i>	<i>I</i>	<i>I</i>	<i>I</i>	<i>W</i>	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

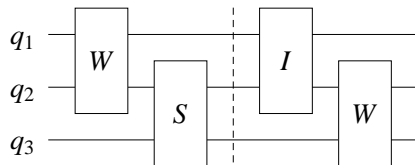


Current configuration

•	•	•	⊙	I	W	S	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

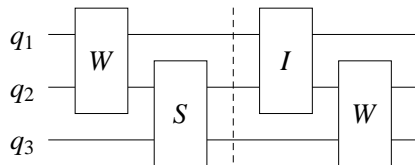


Current configuration

•	•	•	⊙	I	W	S	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

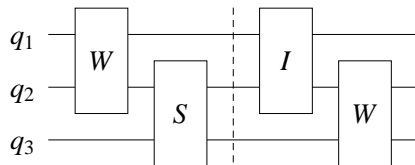


Current configuration

•	•	•	▷	I	W	S	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

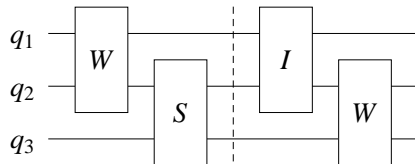


Current configuration

•	•	•	▷	I	W	S	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

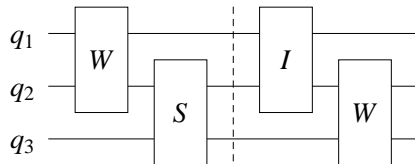


Current configuration

•	•	•	I	\triangleright	W	S	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

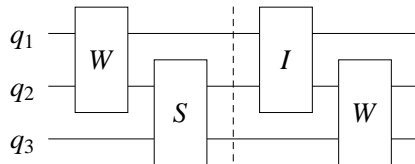


Current configuration

•	•	•	I	W	\triangleright	S	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

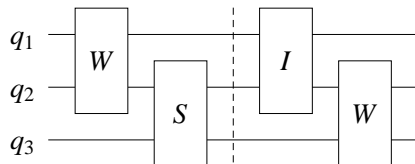


Current configuration

•	•	•	I	W	S	▷	I	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

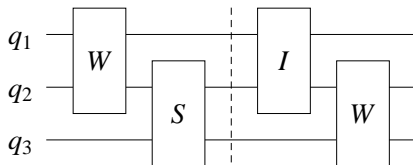


Current configuration

•	•	•	I	W	S	I	\triangleright	I	I	W	•	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

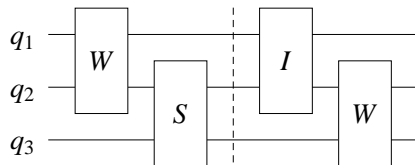


Current configuration

•	•	•	I	W	S	I	I	\triangleright	I	W	•	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

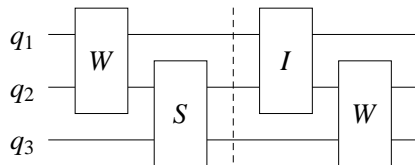


Current configuration

•	•	•	I	W	S	I	I	I	▷	W	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

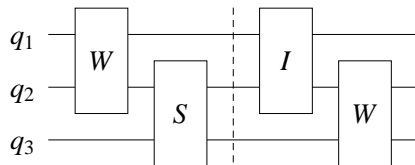


Current configuration

•	•	•	I	W	S	I	I	I	W	▷	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

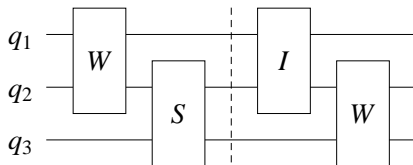


Current configuration

•	•	•	I	W	S	I	I	I	W	▷	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

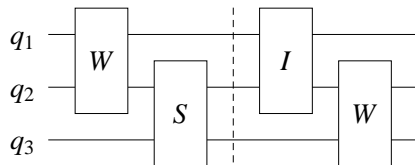


Current configuration

•	•	•	<i>I</i>	<i>W</i>	<i>S</i>	<i>I</i>	<i>I</i>	<i>I</i>	<i>W</i>	⊙	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

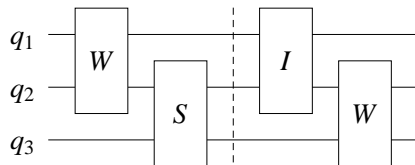


Current configuration

•	•	•	I	W	S	I	I	I	W	⊙	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

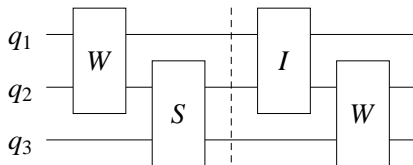


Current configuration

•	•	•	I	W	S	I	I	I	⊗	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

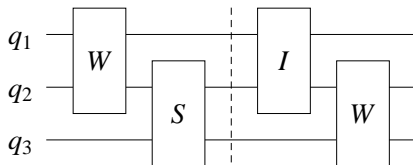


Current configuration

•	•	•	I	W	S	I	I	\textcircled{I}	W	•	•	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

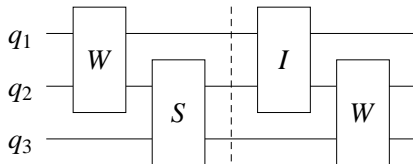


Current configuration

•	•	•	I	W	S	I	\textcircled{I}	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

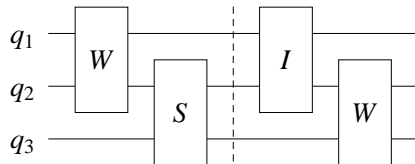


Current configuration

•	•	•	I	W	S	Ⓢ	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

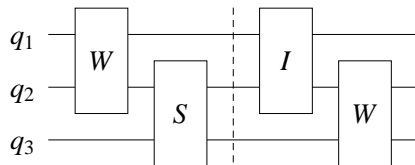


Current configuration

•	•	•	I	W	Ⓢ	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

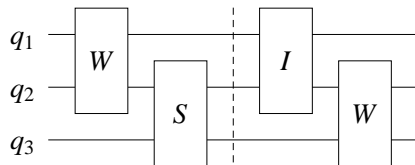


Current configuration

•	•	•	I	Ⓜ	S	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

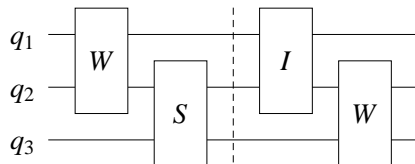


Current configuration

•	•	•	Ⓢ	W	S	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

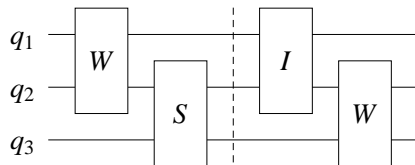


Current configuration

•	•	⊙	I	W	S	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

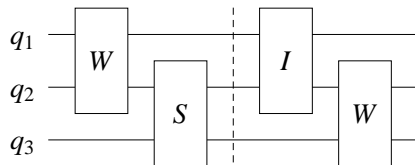


Current configuration

•	•	⊙	I	W	S	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	>	1	0	0	0	1

Evolution

Circuit

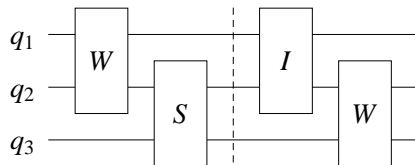


Current configuration

•	•	▷	<i>I</i>	<i>W</i>	<i>S</i>	<i>I</i>	<i>I</i>	<i>I</i>	<i>W</i>	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

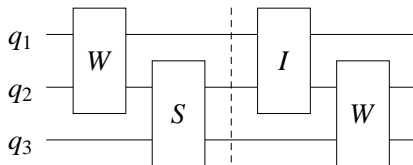


Current configuration

•	•	▷	I	W	S	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

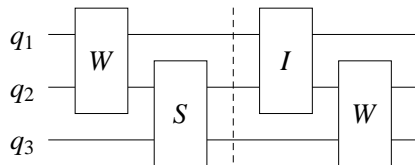


Current configuration

•	•	I	\triangleright	W	S	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

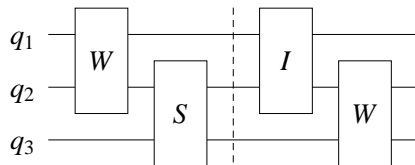


Current configuration

•	•	I	W	\triangleright	S	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

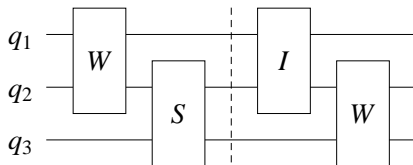


Current configuration

•	•	I	W	S	▷	I	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

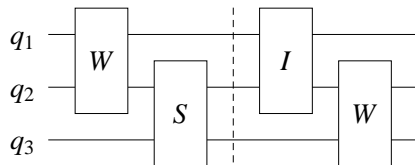


Current configuration

•	•	I	W	S	I	▷	I	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

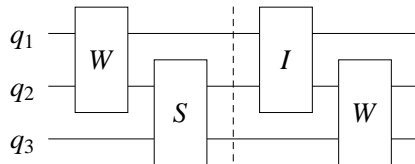


Current configuration

•	•	I	W	S	I	I	\triangleright	I	W	•	•	•	•
0	1	0	0	0	1		q_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

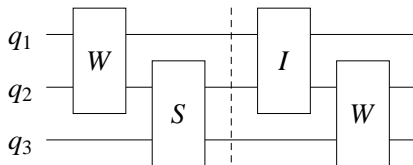


Current configuration

•	•	I	W	S	I	I	I	▷	W	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

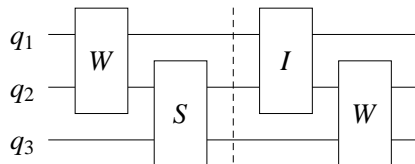


Current configuration

•	•	I	W	S	I	I	I	W	▷	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

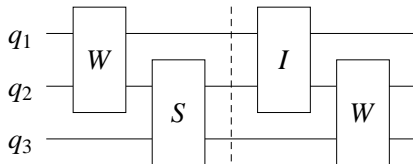


Current configuration

•	•	I	W	S	I	I	I	W	▷	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

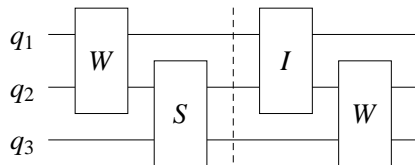


Current configuration

•	•	I	W	S	I	I	I	W	↻	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

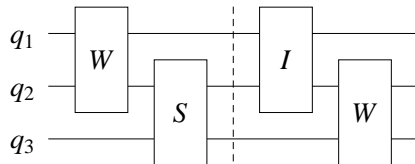


Current configuration

•	•	I	W	S	I	I	I	W	⊙	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

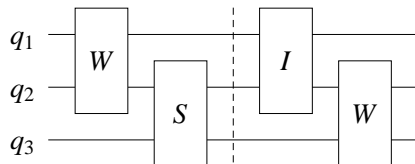


Current configuration

•	•	I	W	S	I	I	I	Ⓜ	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

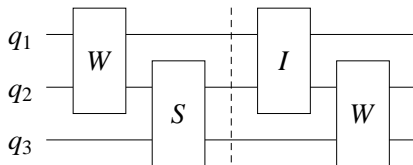


Current configuration

•	•	I	W	S	I	I	\textcircled{I}	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

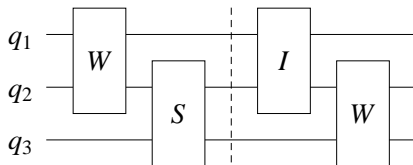


Current configuration

•	•	I	W	S	I	Ⓢ	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

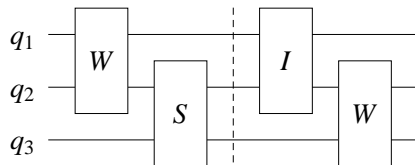


Current configuration

•	•	I	W	S	\textcircled{I}	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

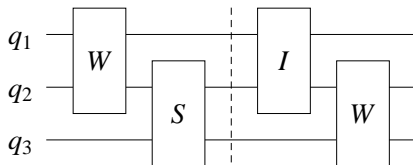


Current configuration

•	•	I	W	Ⓢ	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

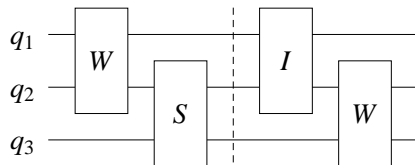


Current configuration

•	•	I	Ⓜ	S	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

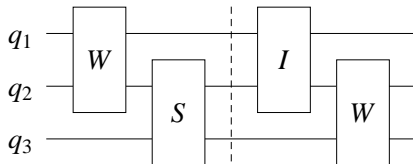


Current configuration

•	•	Ⓢ	W	S	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

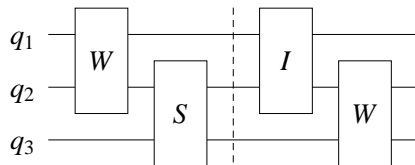


Current configuration

•	⊙	I	W	S	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

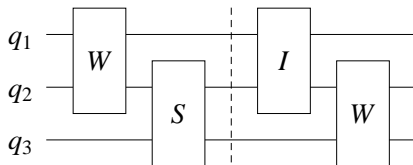


Current configuration

•	⊙	I	W	S	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

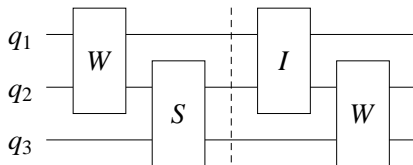


Current configuration

•	▶	I	W	S	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

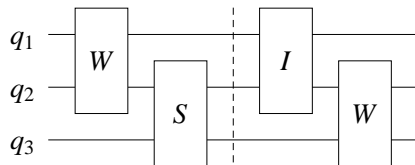


Current configuration

\cdot \blacktriangleright I W S I I I W \cdot \cdot \cdot \cdot \cdot
 0 1 0 0 0 1 $|$ q_{123} \rangle 1 0 0 0 1

Evolution

Circuit

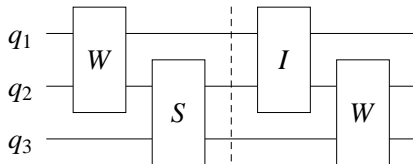


Current configuration

•	I	▶	W	S	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

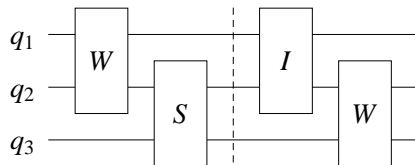


Current configuration

•	I	W	▶	S	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

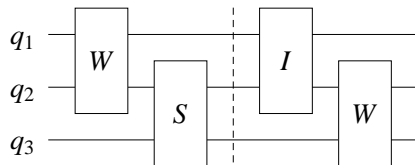


Current configuration

•	I	W	S	▶	I	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

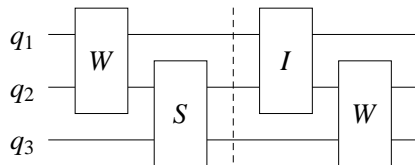


Current configuration

•	I	W	S	I	▶	I	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

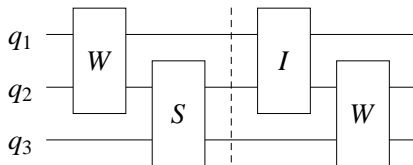


Current configuration

•	I	W	S	I	I	▶	I	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

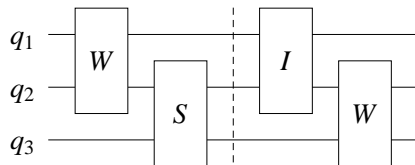


Current configuration

•	I	W	S	I	I	I	▶	W	•	•	•	•	•
0	1	0	0	0	1		q_{123}	⟩	1	0	0	0	1

Evolution

Circuit

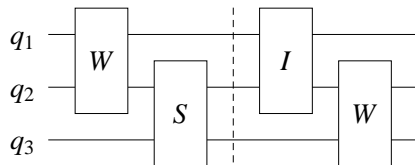


Current configuration

•	I	W	S	I	I	I	W	▶	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

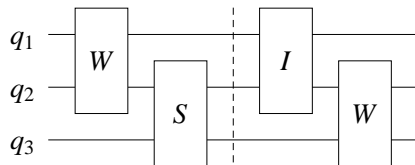


Current configuration

•	I	W	S	I	I	I	W	▶	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

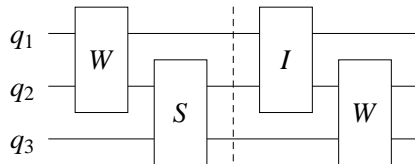


Current configuration

•	I	W	S	I	I	I	W	⊙	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

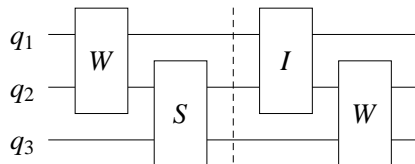


Current configuration

•	I	W	S	I	I	I	W	⊙	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

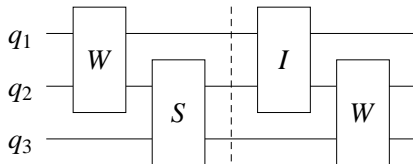


Current configuration

•	I	W	S	I	I	I	\textcircled{W}	•	•	•	•	•	•
0	1	0	0	0	1		s_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

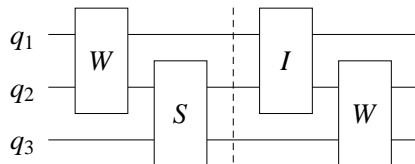


Current configuration

•	I	W	S	I	I	Ⓢ	W	•	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

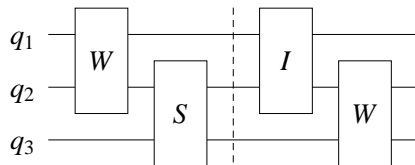


Current configuration

•	I	W	S	I	\textcircled{I}	I	W	•	•	•	•	•	•
0	1	0	0	0	1		s_{123}	\rangle	1	0	0	0	1

Evolution

Circuit

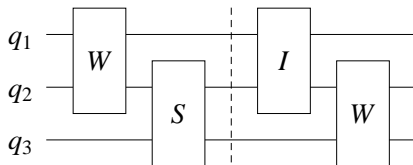


Current configuration

$$\begin{array}{cccccccccccccccc}
 \cdot & I & W & S & \textcircled{I} & I & I & W & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 0 & 1 & 0 & 0 & 0 & 1 & | & s_{123} & \rangle & 1 & 0 & 0 & 0 & 1
 \end{array}$$

Evolution

Circuit

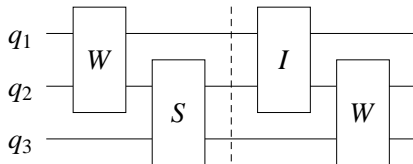


Current configuration

•	I	W	Ⓢ	I	I	I	W	•	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

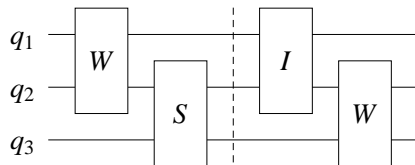


Current configuration

•	I	⊗	S	I	I	I	W	•	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

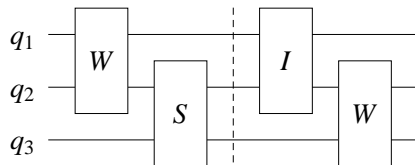


Current configuration

•	Ⓢ	W	S	I	I	I	W	•	•	•	•	•	•
0	1	0	0	0	1		s_{123}	⟩	1	0	0	0	1

Evolution

Circuit

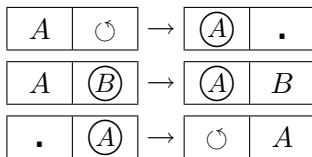


Current configuration

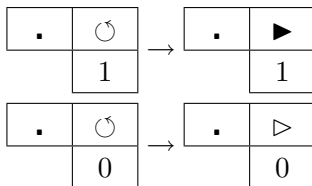
\circlearrowleft	I	W	S	I	I	I	W	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot
0	1	0	0	0	1		s_{123}	\rangle	1	0	0	0	1

Rules 1

Moving the active spot

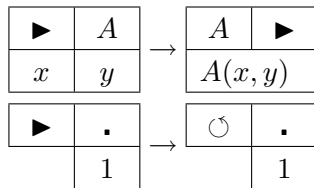


Turning around

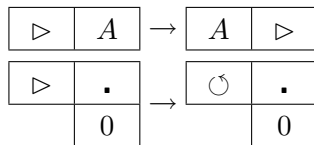


Rules 2

Apply gates



Shift gates



Hamiltonian

These rules give us the following Hamiltonian:

$$H_{20} = - \sum_{j=1}^{L-1} \sum_{r \in \text{rules}} (R_r + R_r^\dagger)_{(j,j+1)}$$

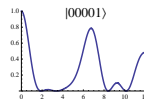
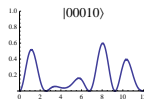
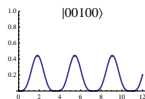
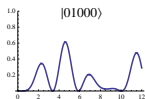
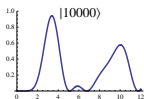
Since the cellular automaton is *deterministic* and *reversible*, we can change the basis so that our Hamiltonian becomes

$$H_{\text{line}} = - \sum_{t=0}^{T-1} (|t\rangle \langle t+1| + |t+1\rangle \langle t|)$$

This is a continuous time quantum walk on a line of length $T + 1$.

“Lonely fermion”

Example with 5 sites and 1 fermion



Conclusion

Conclusion

Hamiltonian QCA

- ▶ on a 1D chain of qudits
- ▶ only nearest-neighbor interactions
- ▶ translationally invariant rules

Conclusion






Hamiltonian QCA

- ▶ on a 1D chain of qudits
- ▶ only nearest-neighbor interactions
- ▶ translationally invariant rules

Evolution

- ▶ diffusion of fermion gas ($d = 10$)
- ▶ quantum walk on a line ($d = 20$)

References

-  Daniel Nagaj, Pawel Wocjan, Hamiltonian Quantum Cellular Automata in 1D, arXiv:0802.0886. Pawel's talk at PI is available at pirsa-08010008.
-  Terry Rudolph, Lov Grover, A 2 rebit gate universal for quantum computing, arXiv:quant-ph/0210187.
-  A. Barenco, C.H. Bennett, R. Cleve, D.P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. Smolin, H. Weinfurter, Elementary gates for quantum computation, arXiv:quant-ph/9503016.
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-  Pictures are taken from <http://ftp.arl.army.mil/ftp/historic-computers/>

Thank you for your attention!



Left: Patsy Simmers, holding ENIAC board
Next: Mrs. Gail Taylor, holding EDVAC board
Next: Mrs. Milly Beck, holding ORDVAC board
Right: Mrs. Norma Stec, holding BRLESC-I board